1

CENTRE OF A CONIC.

The centre of a conic is that point at which all chords of the conic drawn through it are bisected.

THEOREM:

To find the centre of a conic $S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.

Proof:

given eq^n of the conic is

$$S = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \quad \text{--- (1)}$$

let (x_1, y_1) be the centre of the conic. the equation of straight line passes through the point (x_1, y_1) in parametric form is

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = r \quad \text{--- (2)}$$

where, r be the algebraic distance of any point from the fixed point (x_1, y_1)

From (2)

$x - x_1 = lr$

$x = lr + x_1$

& $y - y_1 = mr$

$y = mr + y_1$

Putting the values of x and y in eq (1) we get

$a(lr+x_1)^2 + 2h(lr+x_1)(mr+y_1) + b(mr+y_1)^2 + 2g(lr+x_1) + 2f(mr+y_1) + c = 0$

or, $a(l^2r^2 + x_1^2 + 2lrx_1) + 2h(mlr^2 + lry_1 + mrx_1 + x_1y_1) + b(m^2r^2 + 2mry_1 + y_1^2) + 2glr + 2gx_1 + 2fmr + 2fy_1 + c = 0.$

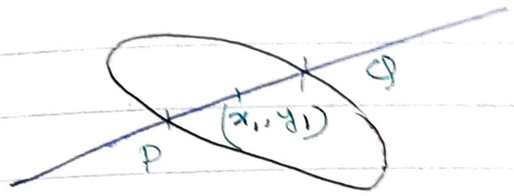
or, $al^2r^2 + ax_1^2 + 2alrx_1 + 2hmlr^2 + 2hlry_1 + 2hmrx_1 + 2hx_1y_1 + bm^2r^2 + 2mbry_1 + by_1^2 + 2glr + 2gx_1 + 2fmr + 2fy_1 + c = 0$

or, $r^2(al^2 + 2hml + bm^2) + 2r(alx_1 + hly_1 + hmx_1 + mby_1 + gl + fm) + (ax_1^2 + 2hx_1y_1 + by_1^2 + 2gx_1 + 2fy_1 + c) = 0$

(3)

or, $r^2(al^2+2hml+bm^2)+2r\{l(ax_1+hy_1+g)+m(hx_1+by_1+f)\}+S_1=0$ ——— (3)

This is quadratic eq^n in r so it has roots say r_1 and r_2.



From above values of r, we get the 2 points of intersection of st. line (11) with the conic (1)

i.e. we get extremities of chord PQ.

But if $c(x_1, y_1)$ be the centre of conic then c must be mid point of PQ

i.e. $CP = CQ$

So, roots of eq^n (3) are equal in magnitude but opposite in signs.

i.e. $r_1 = -r_2$

$r_1 + r_2 = 0.$

i.e. sum of roots of eq^n (3) $= 0$.

or, $-\dfrac{\text{co-efficients of } r}{\text{co-efficient of } r^2} = 0.$

or, $\dfrac{-2\{(ax_1+hy_1+g)l+(hx_1+by_1+f)m\}}{al^2+2hlm+bm^2} = 0.$

or, $(ax_1+hy_1+g)l+(hx_1+by_1+f)m = 0$

identity.

or; Above be an

we have

$ax_1 + hy_1 + g = 0$ —— (4)

$hx_1 + by_1 + f = 0$ —— (5).

for all values of l and m.

Taking cross-multiplication in (4) & (5) we get

$$\frac{x_1}{hf - bg} = \frac{y_1}{hg - af} = \frac{1}{ab - h^2}$$

or, $$x_1 = \frac{hf - bg}{ab - h^2}$$

& $$y_1 = \frac{hg - af}{ab - h^2}$$

$\therefore$ Co-ordinate of centre of conic (1) is

$$\boxed{C(x_1, y_1) = \left(\frac{hf - bg}{ab - h^2}, \frac{hg - af}{ab - h^2}\right)}$$

NOTE:

Since, the centre of conic is obtained on taking cross-multiplications in the eq^n

$$\left.\begin{aligned} ax_1 + hy_1 + g &= 0 \\ \& \quad hx_1 + by_1 + f &= 0 \end{aligned}\right\}.$$

Above eq^n are obtained if we differentiate partially the eq^n.

$S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.

i.e. $\frac{\partial S}{\partial x} = 0$ & $\frac{\partial S}{\partial y} = 0$